## Financial Markets and Management

MiM<br>2020-2021<br>Fall Semester<br>Clara Raposo<br>Problem Set 5 - Guidelines to Solutions<br>Capital Structure - MM's Perfect World<br>Capital Structure - MM with Corporate Taxes<br>Valuation with Corporate Taxes

## TO SOLVE IN CLASS

1. Modigliani-Miller Proposition II

HHH Enterprises is currently an unlevered firm, with an expected return of $10 \%$. It considers a recapitalization through which the firm would get a loan to repurchase its own stock.
a) Suppose HHH borrows so that its debt-equity ratio is 0.75 . With this level of debt, the cost of debt would be $7 \%$. What is the expected return for shareholders after this transaction?
$r_{e}=r_{u}+d / e\left(r_{u}-r_{d}\right)=10 \%+0.75(10 \%-7 \%)=12.25 \%$
b) If however the debt-equity ratio reaches 1.50 , debt will involve more risk and creditors will demand an annual return of $8 \%$. What is the expected return to shareholders, in this case?
$r_{e}=10 \%+1.50(10 \%-8 \%)=13 \%$
2. Modigliani-Miller Proposition II

Suppose that Microsoft has no debt and that its equity cost of capital is $9.5 \%$. The average debt-to-value ratio in the software industry is $14 \%$. What would Microsoft's equity cost of capital be if it chose a level of debt similar to the industry average, with a cost of debt of $6 \%$ ?

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\(r_{E}=r_{U}+\frac{D}{E}\left(r_{U}-r_{D}\right)\)
\(r_{E}=0.095+\frac{0.14}{0.86}(0.095-0.06)\)
    \(=0.1007\)
    \(=10.07 \%\).
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3. Estimate the Present Value of the Interest Tax Shield ( PV(TaxShield) )

Your firm currently has $\$ 100$ million debt at an annual interest rate of $10 \%$. The terms of the loan involve annual repayment of $\$ 20$ million. Suppose that the marginal tax rate that affects this firm is $40 \%$, and that the debt tax shields have the same risk as the loan. What is the present value of the debt tax shields?

| Year | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Debt | 100 | 80 | 60 | 40 | 20 | 0 |
| Interest |  | 10 | 8 | 6 | 4 | 2 |
| Tax Shield | 4 | 3.2 | 2.4 | 1.6 | 0.8 |  |
| PV | 9.67 |  |  |  |  |  |

4. Estimate the Weighted Average Cost of Capital (WACC)

Ragu Inc. currently has debt of \$1 million, and shares with a market capitalization of $\$ 2$ million. The firm pays taxes at $36 \%$, the cost of equity is $10 \%$ and the cost of debt is $7 \%$.
a) Calculate Ragu's pretax WACC rate.

$$
r_{w a c c}=\frac{E}{E+D} r_{E}+\frac{D}{E+D} r_{D}=\frac{2}{3} 10 \%+\frac{1}{3} 7 \%=9 \%
$$

b) What is Ragu Inc.' WACC rate?

$$
r_{\text {wacc }}=\frac{E}{E+D} r_{E}+\frac{D}{E+D} r_{D}\left(1-T_{c}\right)=\frac{2}{3} 10 \%+\frac{1}{3} 7 \%(1-36 \%)=8.16 \%
$$

5. Modigliani-Miller Proposition I with Tc

Millie Industries forecasts an annual FCF of $\$ 6$ million in perpetuity. Its tax rate is $35 \%$ and its unlevered cost of capital is $15 \%$. The firm has debt of $\$ 18$ million, which it intends to keep permanently.
Data:

$$
\begin{gathered}
F C F=\$ 6 m \text { perpetuity } \\
T_{c}=35 \% \\
R_{U}=15 \% \\
D=\$ 18 \mathrm{~m} \text { permanente }
\end{gathered}
$$

a) What would Millie Industries' value be in case it had no debt?

$$
V_{\text {Millie }}^{U}=\frac{\$ 6 m}{0.15}=\$ 40 \text { million }
$$

b) What is Millie Industries' value if financed with debt, as described in the question?

$$
V_{\text {Millie }}^{L}=V_{\text {Millie }}^{U}+T_{c} D=\$ 40 m+0.35 \times \$ 18 \mathrm{~m}=\$ 46.3 \text { million }
$$

6. Valuation with the WACC method

Suppose that Lucky Technologies has an equity cost of capital of $12 \%$, market capitalization of $\$ 12.8$ billion and an enterprise value of $\$ 15.4$ billion. Suppose also that Lucky's cost of debt is $6.5 \%$ and that its marginal tax rate is $35 \%$.
Data:

$$
\begin{gathered}
R_{E}=12 \% \\
E=\$ 12.8 \text { billion } \\
E+D=\$ 15.4 \text { billion } \\
R_{D}=6.5 \% \\
T_{c}=35 \%
\end{gathered}
$$

a) What is Lucky's WACC rate?

$$
r_{W A C C}=\frac{12.8}{15.4} 12 \%+\frac{2.6}{15.4} 6.5 \%(1-0.35)=10.687 \%
$$

b) If Lucky keeps its debt-equity ratio constant, what is the value of a project with average risk and the following expected cash flows?

| Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :--- | :--- | :--- | :--- | :--- |
| FCF | -100 | 70 | 80 | 90 |

$$
\begin{gathered}
P V_{0}=\frac{70}{1.10687}+\frac{80}{1.10687^{2}}+\frac{90}{1.10687^{3}}=194.90 \\
N P V_{0}=-100+194.90=94.90>0 .
\end{gathered}
$$

c) If Lucky keeps its debt-equity ratio as promised, what is the Debt Capacity of the project of question b)?
$D / E=2.6 / 12.8$ constant, or $D /(D+E)=2.6 / 15.4=16.88 \%$ constant, implies that each period $D=0.1688^{*}$ (PV Total)

| Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| FCF | -100 | 70 | 80 | 90 |
| PVt | 194.90 | 145.73 | 81.31 | 0 |
| Dt=0.1688PVI | 32.91 | 24.60 | 13.73 | 0 |

Why?
At $\mathrm{t}=0, \mathrm{PV}=194.90, \mathrm{D}=0.1688^{*} 194.90=32.91$.
At $\mathrm{t}=1$ :
$P V_{1}=\frac{80}{1.10687}+\frac{90}{1.10687^{2}}=145.73$ and $\mathrm{D} 1=0.1688^{*} 145.73=24.60$.
etc...
7. Valuation with the APV method

Consider Lucky's project from question 6.
a) What is Lucky's unlevered cost of capital?

$$
R_{U}=\text { Pre }-\operatorname{tax} W A C C=\frac{12.8}{15.4} 12 \%+\frac{2.6}{15.4} 6.5 \%=11.07 \%
$$

b) What is the unlevered value of the project?

$$
V_{0}^{U}=\frac{70}{1.1107}+\frac{80}{1.1107^{2}}+\frac{90}{1.1107^{3}}=193.55
$$

c) What are the interest tax shields of the project? What is their present value?

| d) Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :---: | :---: | :---: | :---: | :---: |
| Dt=0.1688PVt |  | 24.60 | 13.73 | 0 |
| Interest | 0 | $0.065^{*} 32.91$ | 1.60 | 0.89 |
| =6.5\%*Dt-1 |  | $=2.14$ |  |  |
| ITSt=35\%*Interest | 0 | 0.75 | 0.56 | 0.31 |

$$
P V(I T S)=\frac{0.75}{1.1107}+\frac{0.56}{1.1107^{2}}+\frac{0.31}{1.1107^{3}}=1.36
$$

d) Show that Lucky's project's APV corresponds to the value computed using the WACC method.

$$
V^{L}=V^{U}+P V(I T S)=193.55+1.36=194.90
$$

8. Valuation with the FTE method

Consider Lucky's project of questions 6 and 7 .
a) What are the Free Cash Flows to Equity of the project?

$$
F C F E=F C F-\text { AfterTax InterestPayment }+ \text { NetBorrowing }
$$

| \# | Year | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | FCF | -100 | 70 | 80 | 90 |
| 2 | $\begin{gathered} \text { Interest } \\ =6.5 \%{ }^{*} \mathrm{Dt}-1 \end{gathered}$ | 0 | 2.14 | 1.60 | 0.89 |
| 3 | AfterTax <br> Interest <br> $=$ Interest(1- <br> Tc) | 0 | $\begin{gathered} 2.14(1- \\ 0.35) \\ =1.39 \end{gathered}$ | 1.04 | 0.58 |
| 4 | Dt | 32.91 | 24.60 | 13.73 | 0 |
| 5 | Net <br> Borrowing | 32.91 | $\begin{gathered} 24.60- \\ 32.91= \\ -8.30 \end{gathered}$ | -10.88 | -13.73 |
| $\begin{gathered} 6=1- \\ 3+5 \end{gathered}$ | FCFE | -67.09 | +60.31 | +68.08 | +75.69 |

b) Compute the project's NPV using the Flow to Equity method. Compare it to the NPV computed with the WACC method.

$$
N P V_{0}=-67.09+\frac{60.31}{1.12}+\frac{68.08}{1.12^{2}}+\frac{75.69}{1.12^{3}}=94.90
$$

Same as with WACC method.
9. Changes in the ratio D/E and in the Cost of Capital (Project-Based Cost of Capital)
Santa and Granny (SG) has historically kept a debt-equity ratio of approximately 0.50 . The current share price is $\$ 50$, with 2.5 billion shares outstanding. The firm faces a stable demand; as a consequence it has a low equity beta ( 0.60 ) and can issue debt at a rate of $4.20 \%$, just 20 basis points higher than the risk-free interest rate of $4 \%$. The expected return of the market is $11 \%$ and SG's income tax rate is $30 \%$.

Data:

$$
\begin{gathered}
\frac{D}{E}=0.5 \\
\text { Share Price }=\$ 50 \\
\# \text { Shares }=2.5 \text { billion } \\
\beta_{E}=0.6 \\
R_{D}=4.20 \% \\
R_{F}=4 \% \\
E\left(R_{M}\right)=11 \% \\
T_{C}=30 \%
\end{gathered}
$$

a) This year SG expects a free cash flow of $\$ 6.5$ billion. What constant annual growth rate for the FCF would be consistent with the stock price?
Data:
$F C F_{1}=\$ 6.5$ billion; $g=$ ?

$$
\begin{gathered}
r_{E}=4 \%+0.6(11 \%-4 \%)=8.2 \% \\
r_{W A C C}=\frac{1}{1.5} 8.2 \%+\frac{0.5}{1.5} 4.2 \%(1-0.3)=6.45 \% \\
E=\$ 50 \times 2.5 \text { billion }=\$ 125 \text { billion } \\
D=0.5 E=0.5 \times \$ 125 \text { billion }=\$ 62.5 \text { billion } \\
E+D=\$ 187.5 \text { billion } \\
\$ 187.5 \text { billion }=\frac{\$ 6.5 \text { billion }}{0.0645-g} \\
g=2.98 \%
\end{gathered}
$$

b) SG believes it could increase leverage without creating bankruptcy risk or other costs. With a higher debt-equity ratio of 0.75 , the firm is convinced that the cost of debt would slightly increase to $4.50 \%$. If SG announces that it will implement this restructuring through a leveraged recap, determine the increase in stock price that would result from the additional tax savings.

Data: Leveraged Recapitalization $\frac{D}{E}=0.75 ; R_{D}=4.50 \%$

With Initial Data compute Ru (using MM prop II) or PretaxWacc:

$$
\text { PreTax } W \text { ACC }=\frac{1}{1.5} 8.2 \%+\frac{0.5}{1.5} 4.2 \%=6.87 \%
$$

With the New Capital Structure, re-estimate the levered cost of equity, the wacc rate and the enterprise (present) value of PG:

$$
\begin{aligned}
R_{E} & =6.87 \%+0.75(6.87 \%-4.5 \%)=8.64 \% \\
r_{W A C C} & =\frac{1}{1.75} 8.64 \%+\frac{0.75}{1.75} 4.5 \%(1-0.3)=6.29 \% \\
P V & =\frac{\$ 6.5 \text { billion }}{0.0629-0.0298}=\$ 196.49 \text { billion }
\end{aligned}
$$

Enterprise Value increases by $\$ 8.99$ billion (corresponds to the increment in PV(ITS) due to more debt), which would make the stock price (when the cash of the additional loan entered) rise to:

$$
\text { Stock Price }=\frac{\$ 125 \text { billion }+\$ 8.99 \text { billion }}{2.5 \text { billion }}=\$ 53.6
$$

## 10. Pre-Determined Debt level

On your way to an important meeting regarding an investment project, you review the analysis performed by a trainee in the finance department:

Observing the spreadsheet, you understand that the cash flows are correct, but that your trainee used the FTE method, discounting the FCFE at the firm's equity cost of capital.

|  | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| EBIT |  | 10.0 | 10.0 | 10.0 | 10.0 |
| Interest (5\%) |  | -4.0 | -4.0 | -3.0 | -2.0 |
| Earnings Before Taxes |  | 6.0 | 6.0 | 7.0 | 8.0 |
| Taxes |  | -2.4 | -2.4 | -2.8 | -3.2 |
| Depreciation | -100.0 | 25.0 | 25.0 | 25.0 | 25.0 |
| Cap Ex | -20.0 |  |  |  |  |
| Additions to NWC | 80.0 | 0.0 | -20.0 | -20.0 | -40.0 |
| Net New Debt | -40.0 | 28.6 | 8.6 | 9.2 | 9.8 |
| FCFE | 5.9 |  |  |  |  |
| NPV at 11\% Equity Cost of Capital |  |  |  |  |  |

However, the debt associated with this project differs substantially from the historical debt-equity ratio of the firm (0.40). For this project, the firm will get an upfront Ioan of $\$ 80$ million, repaying $\$ 20$ million in year2, $\$ 20$ million in year 3 , and $\$ 40$ million in year 4 .
Therefore, the equity cost of capital of the project will probably be higher that the firm's and will not be constant over time - invalidating the computations presented in the table above. Clearly the FTE method is not the most appropriate one.

Data:

$$
\begin{gathered}
\frac{D}{E}^{F i r m}=0.4 \\
D_{0}=\$ 80 m ; D_{1}=\$ 80 m ; D_{2}=\$ 60 m ; D_{3}=\$ 40 m ; D_{4}=\$ 0 \\
T_{C}=40 \%
\end{gathered}
$$

a) What is the present value of the interest tax shield associated to this project?

Pre-Determined Debt Levels, $R_{\text {TaxShield }}=R_{D}=5 \%$

| Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Interest=5\%*Dt-1 | 0 | 4 | 4 | 3 | 2 |
| ITSt $=40 \%$ *Interest | 0 | 1.6 | 1.6 | 1.2 | 0.8 |
| $P V($ ITS $)=\frac{1.6}{1.05}+\frac{1.6}{1.05^{2}}+\frac{1.2}{1.05^{3}}+\frac{0.8}{1.05^{4}}=\$ 4.67$ million |  |  |  |  |  |

b) What are the free cash flows of the project?

| FCF = EBIT |  |  |  |  |  |  |  | $(1-T c)+$ Depreciation-CapEx-Increases in NWC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\#}$ | Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |  |  |
| $\mathbf{1}$ | EBIT | 10 | 10 | 10 | 10 |  |  |  |
| $\mathbf{2}$ | EBIT(1-Tc) |  | 6 | 6 | 6 | 6 |  |  |
| $\mathbf{3}$ | Deprec. |  | 25 | 25 | 25 | 25 |  |  |
| $\mathbf{4}$ | CapEx | 100 |  |  |  |  |  |  |
| $\mathbf{5}$ | Change in NWC | 20 |  |  |  | -20 |  |  |
| $\mathbf{6 = 2 + 3 -}$ | FCF | -120 | 31 | 31 | 31 | 51 |  |  |
| $\mathbf{4 - 5}$ |  |  |  |  |  |  |  |  |

c) With the information we have, what is the best estimate of the value of the project?
Using the APV Method:

$$
\begin{gathered}
R_{U}=\frac{1}{1.4} 11 \%+\frac{0.4}{1.4} 5 \%=9.29 \% \\
V^{U}=\frac{31}{1.0929}+\frac{31}{1.0929^{2}}+\frac{31}{1.0929^{3}}+\frac{51}{1.0929^{4}}=\$ 113.81 \text { million } \\
V^{L}=113.81+4.67=\$ 118.48 \text { million } \\
N P V=-120+118.48=-\$ 1.52 \text { million }
\end{gathered}
$$

